DEPARTMENT OF MATHEMATICS
University of Toronto at Mississauga

MAT 133Y Test #4A

March 17, 2006

Fill in the following information in INK!

Last Name: _____.

Given Name: _____.

Student #: _____.

YOUR TUTOR	A.B.	A.C.	D.K.	F.M.	J.H.	R.D.	S.A.	W.N.
YOUR TUTORIAL # OR TUTORIAL DAY/TIME								

Please Circle your Tutor's initials and enter the tutorial number or tutorial day/time

INSTRUCTIONS

1. Only simple scientific calculators are allowed. NO graphing calculators or ones that have memory and are able to manipulate matrices are allowed.
2. The time allotted is **110** minutes.
3. Question 1 is a **MULTIPLE CHOICE** question with 15 parts each worth 3 marks. You must put your answers to the multiple choice questions **IN** the **ANSWER TABLE** provided.
4. For questions 2 and 3 **YOU MUST SHOW ALL YOUR WORK**. A correct answer obtained with false or no reasoning, will **NOT** receive any marks.
5. There are 12 pages to this test, including a "formula sheet". Make sure you have all of them.
6. Budget your time. Good Luck!

Question #	Out of	Score
1	45	
2	30	
3	25	
Total	**100**	

QUESTION 1: MULTIPLE CHOICE

For each question clearly circle the letter corresponding to your answer, in the **MULTIPLE CHOICE ANSWER TABLE provided BELOW** ⇓. (ONLY ONE should be circled for each question).

[Total: 45 marks, 3 marks for each part.

NO marks for incorrect answers or answers indicated outside the MULTIPLE CHOICE ANSWER TABLE]

MULTIPLE CHOICE ANSWER TABLE

1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9	1.10	1.11	1.12	1.13	1.14	1.15
A	A	A	A	A	A	A	A	A	A	A	A	A	A	A
B	B	B	B	B	B	B	B	B	B	B	B	B	B	B
C	C	C	C	C	C	C	C	C	C	C	C	C	C	C
D	D	D	D	D	D	D	D	D	D	D	D	D	D	D
E	E	E	E	E	E	E	E	E	E	E	E	E	E	E

1.1 $\int (x^4 - x)dx =$

A) $4x^3 + C$ B) $x^5 - x^2 + C$ C) $\frac{x^4}{4} - \frac{x^2}{2} + C$ D) $\frac{x^5}{5} - \frac{x^2}{2} + C$ E) None of the above

1.2 $\int e^{4x+3}dx =$

A) $e^{4x+3} + C$ B) $e^{4x} + C$ C) $\frac{1}{4}e^{4x+3} + C$ D) $\frac{e^{4x+4}}{4x+4} + C$ E) None of the above

1.3 The average value $\bar{f}$ of the function $f(x) = \sqrt{x-4}$ on the interval [5, 13] is

A) 2 B) $\frac{1}{6}$ C) $\frac{52}{3}$ D) $\frac{13}{6}$ E) $\frac{13}{4}$

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QUESTION 1: MULTIPLE CHOICE (continued…)

1.4 The demand function for a manufacturer's product is given by $p = 400\text{-}2q$, where p is the price per unit (in dollars) for q units. If its supply equation is $p = q + 100$, then consumers' surplus when market equilibrium is established is

A) \$9000 B) \$9500 C) \$10,000 D) \$10,500 E) \$11,000

1.5 A manufacturer of a product has a marginal cost given by $\frac{dc}{dq} = 0.1q^2 - 20q + 1500$, where c is the total cost (in dollars) of producing q units of a product. If fixed costs are \$30,000, then the total cost of producing 30 units is

A) \$66,600 B) \$66,700 C) \$66,800 D) \$66,900 E) \$67,000

1.6 If $y' = xe^{x^2}$ and $y(0) = \frac{7}{2}$, then $y =$

A) $2e^{x^2} + \frac{3}{2}$ B) $e^{x^2} + \frac{7}{2}$ C) $2e^{x^2} + \frac{7}{2}$ D) $\frac{e^{x^2}}{2} + 3$ E) e^{x^2}

QUESTION 1: MULTIPLE CHOICE (continued…)

1.7 Evaluate $\sum_{k=1}^{4}(2k^2+1)$

A) 60 B) 61 C) 62 D)63 E) 64

1.8 Evaluate $\int_0^3 \frac{4x}{x^2+1}dx =$

A) $\frac{\ln 10}{2}$ B) 0 C) $\ln 10$ D) $2\ln 10$ E) none of the above

1.9 The integral $\int_0^\infty \frac{1}{(x+2)^5}dx =$

A) $\frac{1}{64}$ B) 0 C) $-\frac{1}{16}$ D) $\frac{1}{16}$ E) integral diverges

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QUESTION 1: MULTIPLE CHOICE (continued…)

1.10 The integral $\int_0^{\infty} \frac{1}{\sqrt{2x+1}}\,dx =$

A) 1 B) 0 C) -1 D) $\frac{1}{2}$ E) integral diverges

1.11 If $\frac{dy}{dx} = \frac{e^x}{y^2}$ then $y =$

A) $e^x + C$ B) $\sqrt[3]{\frac{e^x + C}{3}}$ C) $\frac{e^x + C}{3}$ D) $\sqrt{2e^x + C}$ E) $\sqrt[3]{3e^x + C}$

1.12 If X has density function $f(x) = \begin{cases} \frac{2x}{9} & \text{if } 0 \le x \le 3 \\ 0 & \text{otherwise} \end{cases}$, find μ.

A) $\frac{2}{27}$ B) $\frac{1}{9}$ C) $\frac{2}{9}$ D) $\frac{1}{2}$ E) 2

QUESTION 1: MULTIPLE CHOICE (continued…)

1.13 If X is normally distributed with mean $\mu = 150$ and standard deviation $\sigma = 30$, find $P(90<X<180)$.

A) 0.8173 B) 0.1359 C) 0.3185 D) 0.8185 E) 0.8243

1.14 The scores on an examination are normally distributed with mean $\mu = 500$ and standard deviation $\sigma = 100$. What percentage of the scores are less than 360?

A) 8.08% B) 4.192% C) 41.92% D) 80.8% E) none of the above

1.15 Using the trapezoidal rule with $n = 3$ to estimate, the value of $\int_0^3 \frac{1}{x^2+1}\,dx$ is

A) $\frac{1}{3}$ B) $\frac{5}{4}$ C) $\frac{5}{2}$ D) 5 E) none of the above

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QUESTION 2: This question offers you CHOICE! [TOTAL 15 + 15 = 30]
Answer ONLY 2 parts of the following 3 parts (2A, 2B, 2C).

[NOTE: If you write attempt to answer all 3 parts we have the right to choose any 2 of them to be graded.]

PART 2A:

Evaluate: $\int x^4 \ln x dx$. Show all the steps and put your final answer in the box provided.

$\int x^4 \ln x dx =$

QUESTION 2 (continued…): This question offers you CHOICE! [TOTAL 15 + 15 = 30]
Answer ONLY 2 parts of the following 3 parts (2A, 2B, 2C).

[NOTE: If you write attempt to answer all 3 parts we have the right to choose any 2 of them to be graded.]

PART 2B:

The marginal revenue for a company manufacturing x components per week is given by $R'(x) = \dfrac{11(x+7)}{x^2+3x+2}$ where $R(x)$ is the revenue in ten thousands of dollars. Find the equation for $R(x)$. Show all the steps and put your final answer in the box provided.

$R(x) =$

NAME: _____ STUDENT NUMBER: _____

QUESTION 2 (continued…): This question offers you CHOICE! [TOTAL 15 + 15 = 30]
Answer ONLY 2 of the following 3 parts (2A, 2B, 2C).

[NOTE: If you write attempt to answer all 3 parts we have the right to choose any 2 of them to be graded.]

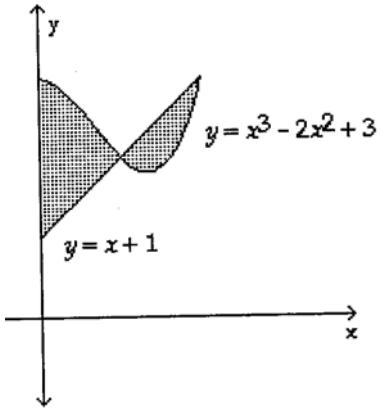
PART 2C:

Evaluate $\int_5^8 \frac{x}{\sqrt{x-4}}dx$. Show all the steps and put your final answer in the box provided.

$\int_5^8 \frac{x}{\sqrt{x-4}}dx$ =

QUESTION 3A: [12 marks]

Express the area *A* of the shaded region in terms of an integral (or integrals). DO NOT EVALUATE your expression. Show all the steps and put your final answer in the box provided.



A =

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QUESTION 3B: [13 marks]

The demand equation for a product is $p = \sqrt{49 - 6q}$ and the supply equation is $p = 1 + q$. Determine the consumers' surplus and the producers' surplus under market equilibrium if p is in dollars. Express your answer to 2 decimals. Show all the steps and put your final answer in the box provided.

CS =

PS =

Integration table:

1. $\int k\,dx = kx + C$
2. $\int u^k\,du = \frac{u^{k+1}}{k+1} + C$ (if k is not equal to -1)
3. $\int u^{-1}\,du = \int \frac{1}{u}\,du = \ln|u| + C$
4. $\int e^u\,du = e^u + C, \quad \int e^{kx}\,dx = \frac{e^{kx}}{k} + C$
5. $\int a^u\,du = \frac{a^u}{\ln a} + C$
6. $\int cf(x)\,dx = c\int f(x)\,dx$
7. $\int \big(f(x) \pm g(x)\big)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
8. $\int f(g(x))g'(x)\,dx = \int f(u)\,du$, where $u = g(x)$

[this is the 'reverse' of the chain rule of differentiation and is known as the "substitution method for integration"]

Some useful sums that occur often are:

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} \qquad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

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Trapezoidal Rule:

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\big[f(a) + 2f(a+h) + 2f(a+2h) + \ldots + 2f(a+(n-1)h) + f(b)\big]$$

where $h = \frac{b-a}{n}$.

Simpson's Rule:

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\big[f(a) + 4f(a+h) + 2f(a+2h) + \ldots + 4f(a+(n-1)h) + f(b)\big]$$

where $h = \frac{b-a}{n}$ and n is even.

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-*Point of equilibrium* (q_0, p_0): Where the demand curve and the supply curve intersect.

-*CS* Consumers' surplus under market equilibrium is geometrically represented by the area of the region ABOVE $p = p_0$ and below the demand curve, from $q = 0$ to $q = q_0$

- *PS* Producers' surplus under market equilibrium is geometrically represented by the area of the region BELOW $p = p_0$ and above the supply curve, from $q = 0$ to $q = q_0$

For continuous random variables with distribution $f(x)$, we have the following:

The mean or expected value is:

$$\mu = E(x) = \int_{-\infty}^{\infty} xf(x)\,dx$$

The variance is:

$$\sigma^2 = Var(x) = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)\,dx$$

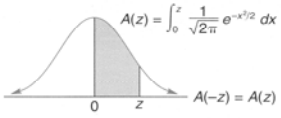
This can also be calculated as:

$$\sigma^2 = Var(x) = \int_{-\infty}^{\infty} x^2 f(x)\,dx - \mu^2$$

and the standard deviation is:

$$\sigma = \sqrt{Var(x)}$$

974 **Appendix D** ▸ *Areas Under the Standard Normal Curve*



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	.1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	.4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	.4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	.4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990
3.1	.4990	.4991	.4991	.4991	.4992	.4992	.4992	.4992	.4993	.4993
3.2	.4993	.4993	.4994	.4994	.4994	.4994	.4994	.4995	.4995	.4995
3.3	.4995	.4995	.4995	.4996	.4996	.4996	.4996	.4996	.4996	.4997
3.4	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4997	.4998
3.5	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998	.4998